6.4110
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1. You're trying to use first-order logic to model what happens when you sterilize a jar. You think that sterilization will kill all the bacteria that are not resistant. You formalize this in first-order logic as

$$\forall x. B(x) \land \neg R(x) \to D(x) \tag{1}$$

where B means "is a bacterium", R means "is resistant" and D means "is dead."

You observe (through a microscope!) that there is at least one live bacterium in the jar:

$$\exists y. B(y) \land \neg D(y). \tag{2}$$

You are now wondering whether there are any resistant bacteria:

$$\exists z. B(z) \land R(z). \tag{3}$$

In other words, you want to know whether sentences (1) and (2) entail sentence (3).

(a) Recall that a logic *model* is composed of a *universe* of objects and an *interpretation* that maps predicates to object relations.

i. If you found a model in which sentence (1) is true and sentence (3) is false, would that guarantee that (1) and (2) do not entail (3)? ◯ Yes ◯ No

ii. If you found a model in which sentence (1) is true, sentence (2) is true, and sentence (3) is false, would that guarantee that (1) and (2) do not entail (3)? ◯ Yes ◯ No

iii. If you found a model in which sentence (1) is false and sentence (3) is true, would that guarantee that (1) and (2) do not entail (3)? ◯ Yes ◯ No

$$\forall x. B(x) \land \neg R(x) \to D(x) \tag{1}$$
$$\exists y. B(y) \land \neg D(y) \tag{2}$$
$$\exists z. B(z) \land R(z) \tag{3}$$

(b) Convert sentences (1) and (2) to clausal form.

(c) Use resolution refutation to prove that (1) and (2) entail (3).

(d)

$$\forall x.B(x) \land \neg R(x) \rightarrow D(x) \quad (1)$$

$$\exists y.B(y) \land \neg D(y) \quad (2)$$

$$\exists z.B(z) \land R(z) \quad (3)$$

Pat sees your logic sentences (1-3) scribbled on a napkin and, unaware of the bacteria story, wonders about the following logic model:

- The universe $\mathcal{U}$ = the set of real numbers
- The interpretation $\mathcal{I}$ is:

$$\mathcal{I}(B) = \text{the set of integers}$$
$$\mathcal{I}(R) = \{x : x > 0\}$$
$$\mathcal{I}(D) = \{x : x < 1\}$$

i. Is sentence (1) true in this model? ◯ Yes ◯ No
Explain your reasoning.

ii. Is sentence (2) true in this model? ◯ Yes ◯ No
Explain your reasoning.

iii. Is sentence (3) true in this model? ◯ Yes ◯ No
Explain your reasoning.