Modeling Practice

1 Bridgerton Regency Seating

1.1 Problem Setup

Guests. Let G be a finite set of guests. Each guest $g \in G$ has:

$$\mathrm{rank}(g) \in \mathbb{N} \quad \text{(larger means more important)}$$

$$\mathrm{marital}(g) \in \{\text{Married, Fiance, Debutante or Eligible Gentleman, Spinster, Child}\}$$

$$\mathrm{partner}(g) \in G \cup \{\varnothing\}$$

$$\mathrm{gender}(g) \in \{M, F\}$$

Seats. Let S be a finite set of labeled *guest* seats. We assume a rectangular table with the host and the hostess seated at the two ends (these end seats are fixed and are not included in S).

The table has N guest seats on each side, for a total of $2N$ guest seats labeled

$$S = \{1, \ldots, 2N\}.$$

Seats are arranged so that even-numbered seats lie on the bottom and the odd-numbered seats lie on the top (see Figure 1).

Figure 1: Visualizing the seating constraints (excluding Social Condition). Rank flows from the ends (high) to the center (low). Adjacent and diagonal spacing of couples is forbidden. Genders alternate along the sides.

1.2 CSP Formulation

Constraints. Our goal is to specify each constraint $c_{i,j} \in C$ as a pair $((\text{scope}), R)$ where R is a relation of allowed tuples.

1. **Possible assignment (no guest used twice).** Each seat is assigned exactly one guest, and no guest appears in more than one seat.

2. **Seats of honor.** The highest-precedence gentleman is placed at the hostess's right (seat $2N - 1$), and the highest-precedence lady is placed at the host's right (seat 2).

3. **Social conditions.** Starting from the seats nearest the host and hostess and moving inward along each side toward the middle of the table, precedence must not increase.

4. **Alternation of genders.** Along each side of the table, adjacent seats should alternate male/female.

5. **Couple separation.** Married couples and fiancés should not be seated together (adjacent, across, or diagonal).

2   Factory Problem from Class

Factory problem: smaller

Objects:
- Machines: sander, painter, dryer
- Parts: A, B
- Times: $1, \dots, 4$

Constraints:
- Each part must be sanded before painted before dried.
- The sander and painter can each operate on at most one part at a time.
- The sander can't operate at the same time the dryer is operating.

One formulation: Variables: pm: when is part p in machine m? Domain of variables are times.

Figure 2: Factory Problem From Class

2 CSP

2.1 CSP Review

1. **Constraint Graph** Graph where nodes correspond to variables of a problem and edges connect any two variables participating in a constraint.
2. **Arc Consistency** A variable in a CSP is arc-consistent if every value in its domain satisfies the variable's constraints. A graph is arc-consistent if every variable is arc-consistent with every other variable

2.2 CSP Practice: 4-Queens problem

Consider a 4x4 chess-board. Place four queens on the board such that no two of the queens can attack each other. Queens can attack other queens placed in the same row, column, or along the same diagonal

For example, if we place a queen on (2,2), the following squares can not be occupied by a queen.

Since we know that each column can only have one queen, let us represent this problem with four variables, X_1, X_2, X_3, X_4, where X_i represents the row of the queen in column i. In the example above, $X_2 = 2$. The domain of each X_i is $\{1, 2, 3, 4\}$.
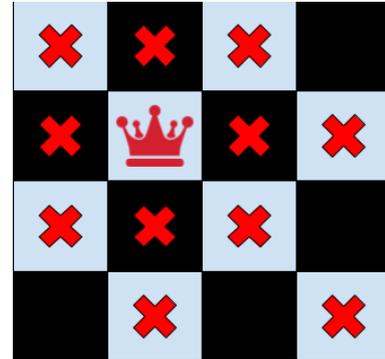
We can represent this using the following constraint graph. Each edge in the graph represents a constraint that the two queens connected by the edge may not be in the same row and that two queens may not be along the same diagonal.

X1
{1,2,3,4}

X2
{1,2,3,4}

X3
{1,2,3,4}

{1,2,3,4}
X4
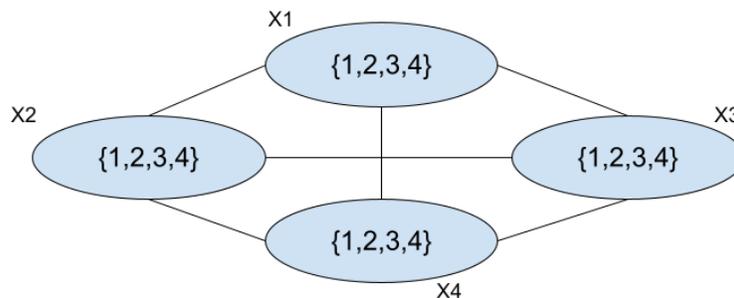
2.2.1 Problem 1

Execute the AC-3 algorithm with the additional constraint that $X_1 = 1$. Does a solution exist? How can you tell?

2.2.2 Problem 2

Execute backtracking with forward-checking to find a solution to the above CSP. Assign variables in the order X_1, X_2, X_3, X_4 and the values from 1 to 4.