6.4110/16.420
Representation, Inference and Reasoning in AI

Quiz 1 Practice A

Solutions

Feburary 16, 2026

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

You are permitted to use a single sheet of paper with notes on (both sides), and a calculator and a timer. You may not use your phone as a calculator. **Box all answers** for free response questions.

Name: ______________________________

MIT email: ______________________________

Question	Points	Score
1	45	
2	40	
3	15	
Total:	100	

1 Bridgerton Regency Seating

This question is adapted from PSS1. See PSS1 for the full problem.

1.1 Problem Setup

Guests. Let G be a finite set of guests. Each guest $g \in G$ has:

$$\text{rank}(g) \in \mathbb{N} \quad \text{(larger means more important)}$$
$$\text{marital}(g) \in \{\text{Married}, \text{Fiance}, \text{Debutante or Eligible Gentleman}, \text{Spinster}, \text{Child}\}$$
$$\text{partner}(g) \in G \cup \{\varnothing\}$$
$$\text{gender}(g) \in \{M, F\}$$

Seats. Let S be a finite set of labeled guest seats. We assume a rectangular table with the host and the hostess seated at the two ends (these end seats are fixed and are not included in S).
The table has N guest seats on each side, for a total of 2N guest seats labeled

$$S = \{1, \ldots, 2N\}.$$

Seats are arranged so that even-numbered seats lie on the bottom and the odd-numbered seats lie on the top (see Figure 1).

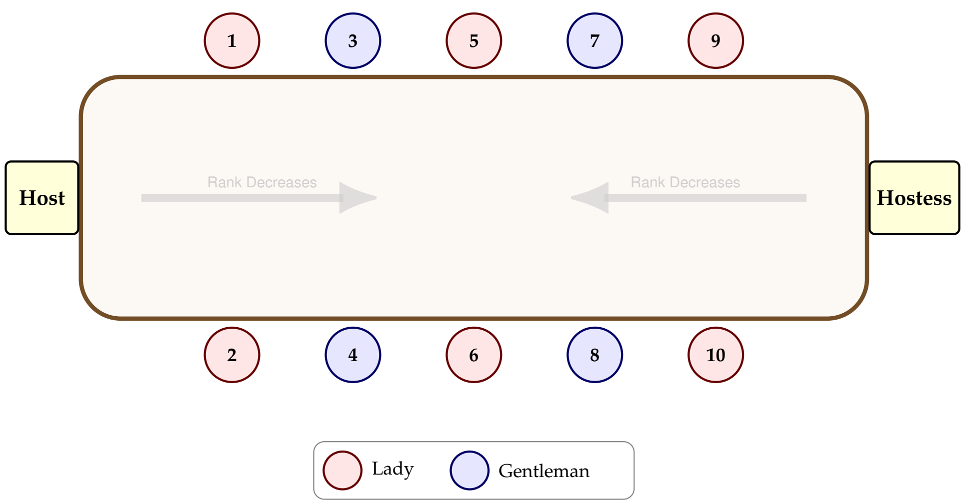


Figure 1: Visualizing the seating constraints (excluding Social Condition). Rank flows from the ends (high) to the center (low). Adjacent and diagonal spacing of couples is forbidden. Genders alternate along the sides.

1. ## 1.2 CSP Formulation

Solution:

Variables and domains. For each seat $i \in S = \{1, \ldots, 2N\}$ define a variable X_i whose value is the guest seated at i:

$$X_i \in G.$$

Constraints. Our goal is to specify each constraint $c_{i,j} \in C$ as a pair $\big((\text{scope}), R\big)$ where R is a relation of allowed tuples.

(a) (10 points) **Possible assignment (no guest used twice).** Each seat is assigned exactly one guest, and no guest appears in more than one seat.

Solution:

$$R^{\neq} = \{(g, h) \in G \times G \mid g \neq h\}.$$

$$c^{\neq}_{i,j} = \Big((X_i, X_j),\ R^{\neq}\Big), \qquad 1 \leqslant i < j \leqslant 2N.$$

(b) (25 points) **Social conditions.** Starting from the seats nearest the host and hostess and moving inward along each side toward the middle of the table, rank must not increase. For people of the same rank, marital status must not increase (assume married is the highest rank, and it decreases from left to right in the set definition in the problem setup).

Solution: Fix a numeric precedence for marital condition:

$$\pi : \{\text{Married}, \text{Fiance}, \text{Debutante or Eligible Gentleman}, \text{Spinster}, \text{Child}\} \to \{4, 3, 2, 1, 0\}$$

with larger meaning higher precedence, and define

$$\text{key}(g) := \big(\text{rank}(g),\ \pi(\text{marital}(g))\big).$$

Fix a numeric precedence for marital condition: Define lexicographic order $\succeq$ on keys by

$$(a, b) \succeq (c, d) \Leftrightarrow \big(a > c\big)\ \vee\ \big(a = c \wedge b \geqslant d\big).$$

Define the binary precedence relation

$$R^{\succeq} = \{(g, h) \in G \times G \mid \text{key}(g) \succeq \text{key}(h)\}.$$

Let the middle column be

$$m := \left\lceil \frac{N}{2} \right\rceil.$$

$$\forall i \in \{1, \ldots, m-1\}: \quad c_i^{\succeq_1} = \big((X_i, X_{i+1}), R^{\succeq}\big) \quad \text{and} \quad c_i^{\succeq_2} = \big((X_{2N+1-i}, X_{2N-i}), R^{\succeq}\big).$$

(c) (10 points) **Alternation of genders.** Along each side of the table, adjacent seats should alternate male/female.

Solution:

$$R^{\text{alt}} = \{(g, h) \in G \times G \mid \text{gender}(g) \neq \text{gender}(h)\}.$$

$$\overline{R^{alt}} = \{G \times G\} - R^{alt}.$$

$$\forall i \in \{1, 3, \dots, 2N-1\}: \quad c_i^{\overline{alt}} := \big((X_i, X_{i+1}), \overline{R^{alt}}\big),\ c_i^{alt} := \big((X_i, X_{i+2}), R^{alt}\big).$$

$c_i^{\overline{alt}}$ ensures people sitting across the table are the same gender, while c_i^{alt} ensures people sitting to the left or right are different genders. This holds for both sides by transitivity.

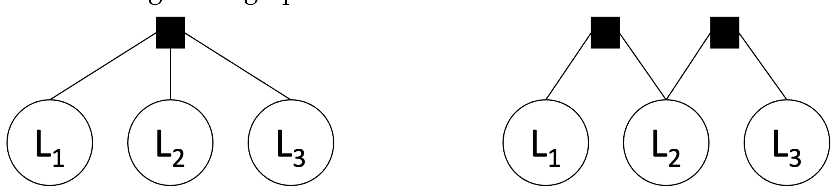
2 Purple Loosestrife

2. Purple Loosestrife (PL) is an invasive plant species. You are an ecologist trying to eradicate it. But before you can do that you need to figure out where it is likely to be.

(a) Let's start with a simple world, with just three locations, L_1, L_2, L_3, connected in a line. We will model each one with a random variable that has possible values 0 (no PL) and 1 (PL). You are told that

- $P(L_2 = 1 \mid L_1 = 1, L_3 = 1) = 0.9$
- $P(L_2 = 1 \mid L_1 = 0, L_3 = 0) = 0.1$
- $P(L_2 = 1 \mid L_1 = 0, L_3 = 1) = 0.5$
- $P(L_2 = 1 \mid L_1 = 1, L_3 = 0) = 0.5$

Consider the following factor graphs:

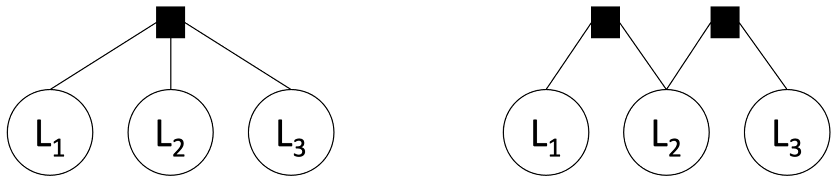


i. (10 points) Provide a table $\phi_{1,2,3}$ that causes the factor graph on the left to represent a distribution consistent with the constraints above. Is there more than one such table?

Solution:

L_1	L_2	L_3	V
0	0	0	9
0	0	1	3
0	1	0	1
0	1	1	3
1	0	0	3
1	0	1	1
1	1	0	3
1	1	1	9

Yes, there are multiple tables, because we don't care about normalization.



ii. (5 points) Kris considers using the factor graph on the right, and actually using the same table for both factors. Which of the following is true?

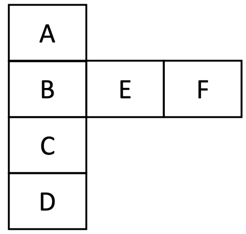
- √ **The factor graph on the left can represent more distributions than Kris's model**
- ○ The factor graph on the left can represent fewer distributions than Kris's model
- ○ The factor graph on the left can represent the same set of distributions as Kris's model

iii. (10 points) Lucky for Kris, there exists at least one table $\phi_{12} = \phi_{32}$ that causes the factor graph on the right to represent a distribution consistent with the conditional probability constraints we started with. Provide such a table.

Solution:

A	B	V
0	0	3
0	1	1
1	0	1
1	1	3

(b) Now let's consider a larger environment with six locations: A, B, C, D, E, and F. Here is a map:



We will again consider a factor graph with variables corresponding to locations. This time, we will create a single pairwise factor between each pair of locations that are connected in the map. Moreover, the potential for each factor is the same:

	X = 0	**X = 1**
Y = 0	10	1
Y = 1	1	10

where X and Y are any adjacent locations. You are doing belief propagation and get messages

$$\mu_{\phi_{AB} \to B} = (1, 2) \qquad \mu_{\phi_{CB} \to B} = (2, 3)$$

i. (5 points) What is $\mu_{B \to \phi_{BE}}$?

Solution:
$[2, 6]$

ii. (10 points) What is $\mu_{\phi_{BE} \to E}$?

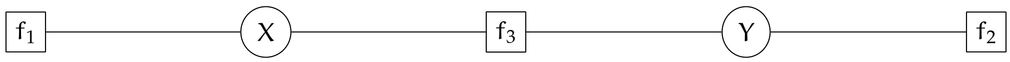
Solution:

B	E	V
0	0	10 * 2 = 20
0	1	1 * 2 = 2
1	0	1 * 6= 6
1	1	10 * 6 = 60

Now marginalize out B to get $(26, 62)$.

3 Factor Graph

3. Consider two random variables X and Y, both defined over the same domain $\mathcal{S}$. Their joint distribution can be represented as the following factor graph.



In this figure, rectangular nodes are factors; circular nodes are random variables.

(a) (5 points) Write a formula for the joint distribution of $p(X = x, Y = y)$, where $x \in \mathcal{S}$, $y \in \mathcal{S}$, represented as a function of the factor potentials f_1, f_2, and f_3.

Solution:

$$p(X = x, Y = y) = \frac{f_1(x)f_2(y)f_3(x, y)}{\sum_{x' \in X} \sum_{y' \in Y} [f_1(x')f_2(y')f_3(x', y')]}$$

(b) (5 points) Write a formula for the conditional distribution of $p(X = x|Y = y)$, represented as a function of the factor potentials f_1, f_2, and f_3.

Solution:

$$p(X = x|Y = y) = \frac{f_1(x)f_2(y)f_3(x, y)}{\sum_{x' \in X} [f_1(x')f_2(y)f_3(x', y)]}$$

(c) (5 points) Consider the following claim. For any $x \in S$:

Claim 1.

$$\sum_y p(X = x|Y = y) = p(X = x).$$

Prove the claim above or provide a counter-example.

Solution: The claim is false.
Consider the following joint distribution, defined over $x, y \in S = \{0, 1\}$.

$$p(X = 0, Y = 0) = 0.5, p(X = 1, Y = 0) = 0, p(X = 0, Y = 1) = 0.5, p(X = 1, Y = 1) = 0.$$

Thus,

$$p(X = 0|Y = 0) = 1, p(X = 0|Y = 1) = 1.$$

It's not even a distribution.