6.4110/16.420
Representation, Inference and Reasoning in AI

Quiz 1

February 25, 2026

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

You are permitted to use a single sheet of paper with notes on (both sides). You may not use a calculator. **Box all answers** for free response questions.

Name: ______________________________

MIT email: ______________________________

Question	Points	Score
1	27	
2	39	
3	34	
Total:	100	

6.4110/16.420
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McDolphin

1. Assume you have a set of employees $e \in E$, restaurant locations $l \in L$ within a town, and discrete time slots $h \in H$ over a week. Each employee e has a set of locations they are able to work at $L_e \subseteq L$ and a set of time slots during which they are available $H_e \subseteq H$. Each location l requires staffing during a specified subset of time slots $H_l \subseteq H$. A location is staffed at a given time if **exactly one** employee is working there, though an employee could work more than one hour at more than one location total. Your goal is to find a staffing schedule so that every location is staffed at each hour it is open.

   (a) For each of the three possible formulations below, indicate the domains of the variables so that an assignment between **each variable and a single element** of the domain would constitute a possible solution to the problem. No formulation should place additional constraints on valid assignments beyond those described above. If the answer is "None of these," explain in the box below, select the valid domain, and leave the box blank.

      i. (3 points) One variable $V_{l,h}$ for each location $l \in L$ and hour $h \in H_l$

      ○ E ○ H ○ L ○ $H \times L$ ○ $H \times E$ ○ $E \times L$ ○ None of these (Explain below)

      ii. (3 points) One variable V_l for each location $l \in L$.

      ○ E ○ H ○ L ○ $H \times L$ ○ $H \times E$ ○ $E \times L$ ○ None of these (Explain below)

iii. (3 points) One variable $V_{l,e}$ for each employee $e \in E$ and location $l \in L_e$.
○ E ○ H ○ L ○ $H \times L$ ○ $H \times E$ ○ $E \times L$ ○ None of these (Explain below)

(b) For the problems below, express the constraints using these variables:

$$E, H, L, L_e, H_e, H_l, e, h, l, V_{l,h}, \textit{Pat}\ (P), \textit{Plainville (N)}, \text{ and } \textit{3pm Thursdays}\ (3pmT).$$

If you find it useful, you may introduce additional variables or domains derived from these, but you must clearly and explicitly define anything new you use. Every answer must be written in set notation; we will allow some flexibility in how you specify the sets, provided they are clearly and correctly defined.

i. (4 points) In the formulation with one variable $V_{l,h}$ for each location and hour, what are the unary constraints? Recall that if a constraint is already satisfied by variable specification, it should not be a constraint.

ii. (4 points) In the formulation with one variable $V_{l,h}$ for each location and hour, we need to specify some binary constraints. For two variables V_{l_1,h_1} and V_{l_2,h_2}, provide an expression for the binary constraint between them.

iii. (4 points) What constraint would you add to represent that an employee Pat is unable to staff Plainville at 3PM on Thursdays?

(c) (6 points) Consider a problem instance with 2 locations (**P**lainville, **C**ambridge), 2 hours (**8am** and **6pm** on Monday), and 3 employees (**A**lex, **B**laine, and **D**elilah), where both locations are open during both hours. Assume the problem formulation where we have variables $V_{l,h}$ for each location and hour. All three employees are students at MIT.

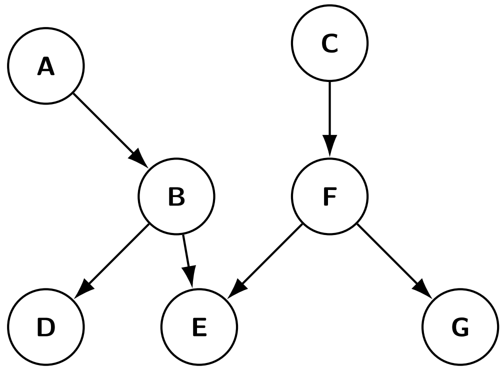
1. **A** takes classes in the afternoon, so could only work at **P** in the morning, but could make it to the location in **C** at any time of day.
2. Since **B** never wakes up before 1pm she is unable to work in the morning.
3. **A** babysits **B's** pet fish so can't work during any time that **B** is working.
4. **D** is very flexible on time and location but is only willing to work a single hour a week because she is hosed with problem sets.

Using the formulation with one variable per location,hour pair, assuming variable ordering $V_{\text{C,6pm}}$, $V_{\text{C,8am}}, V_{\text{P,6pm}}, V_{\text{P,8am}}$, if we assign **B to work at C at 6pm**, which possible assignments does forward checking eliminate? Respond by listing the contents of the domain for each variable before and after this assignment.

Variable	**Domain (before assigning** B **to** $V_{C,6PM}$**)**	**Domain (after assigning** B **to** $V_{C,6PM}$ **and FC that assignment)**
$V_{\text{C,6pm}}$	________________	________________
$V_{\text{C,8am}}$	________________	________________
$V_{\text{P,6pm}}$	________________	________________
$V_{\text{P,8am}}$	________________	________________

Fun with Factors

2. Consider the following Bayesian network:



(a) (2 points) Is it a polytree?
◯ Yes ◯ No

(b) (3 points) Is it possible to express any joint distribution on these 7 variables using this network structure?
◯ Yes ◯ No

(c) (3 points) Imagine that we just found out the value of D and A. Does this tell us anything about C?
◯ Yes ◯ No

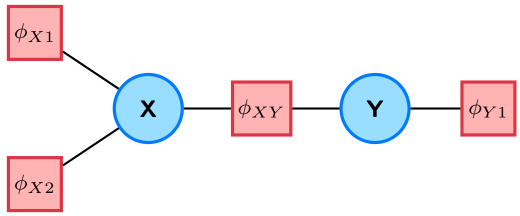
(d) (3 points) Say we already know the value of F. If we additionally learn the value of E, does this tell us anything more about what C might be?
◯ Yes ◯ No

(e) (3 points) Say we learned the value of A. Given this information, is B independent of F?
◯ Yes ◯ No

(f) (3 points) Select all variables that, if observed, would make A and C dependent.
◯ A ◯ B ◯ C ◯ D ◯ E ◯ F ◯ G ◯ There is no such variable

(g) (6 points) Draw the Bayesian network above as an undirected factor graph.

(h) (6 points) Consider a new factor graph with two binary variables, $X \in \{0, 1\}$ and $Y \in \{0, 1\}$. The graph contains one pairwise factor ϕ_{XY} connecting them, two unary factors ϕ_{X1} and ϕ_{X2} connected to X, and one unary factor ϕ_{Y1} connected to Y.



The factor tables are defined as follows:

X	$\phi_{X1}(X)$
0	2
1	1

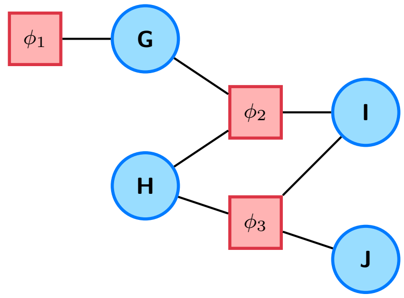
X	$\phi_{X2}(X)$
0	1
1	3

Y	$\phi_{Y1}(Y)$
0	3
1	1

X	Y	$\phi_{XY}(X, Y)$
0	0	1
0	1	2
1	0	2
1	1	1

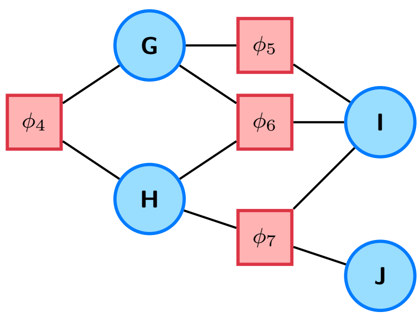
Calculate the exact marginal probabilities $P(X)$ and $P(Y)$.

(i) (10 points) Quinn is analyzing the factor graph below, but realizes that exact Belief-Propagation can't work since there is a loop.



For each of the following modified factor graphs, indicate whether it could be made **equivalent** to the original graph with a careful selection of new factors, and whether exact **Belief-Propagation** could be successfully run on it.

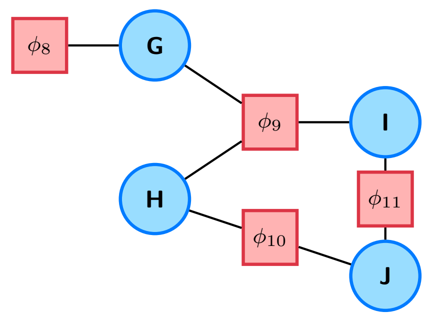
Graph A



Distribution:
◯ Could be Equivalent ◯ Not Equivalent

Exact BP Works?
◯ Yes ◯ No

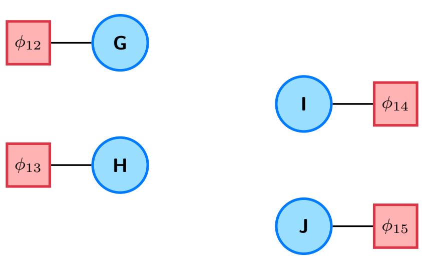
Graph B



Distribution:
◯ Could be Equivalent ◯ Not Equivalent

Exact BP Works?
◯ Yes ◯ No

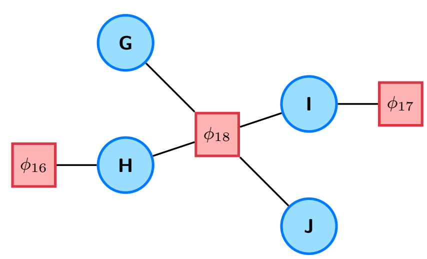
Graph C



Distribution:
◯ Could be Equivalent ◯ Not Equivalent

Exact BP Works?
◯ Yes ◯ No

Graph D



Distribution:
◯ Could be Equivalent ◯ Not Equivalent

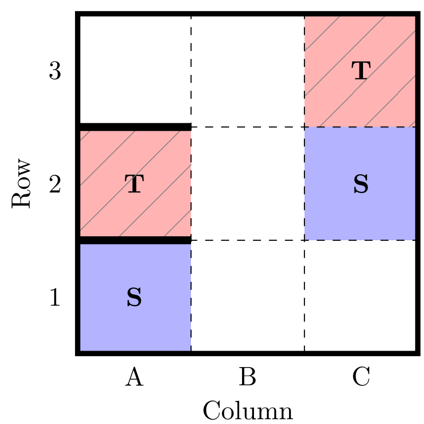
Exact BP Works?
◯ Yes ◯ No

Dungeon Crawler

3. You are playing a text-based RPG called *Dungeon Crawler*. The game is notoriously unfair and relies entirely on random moves.

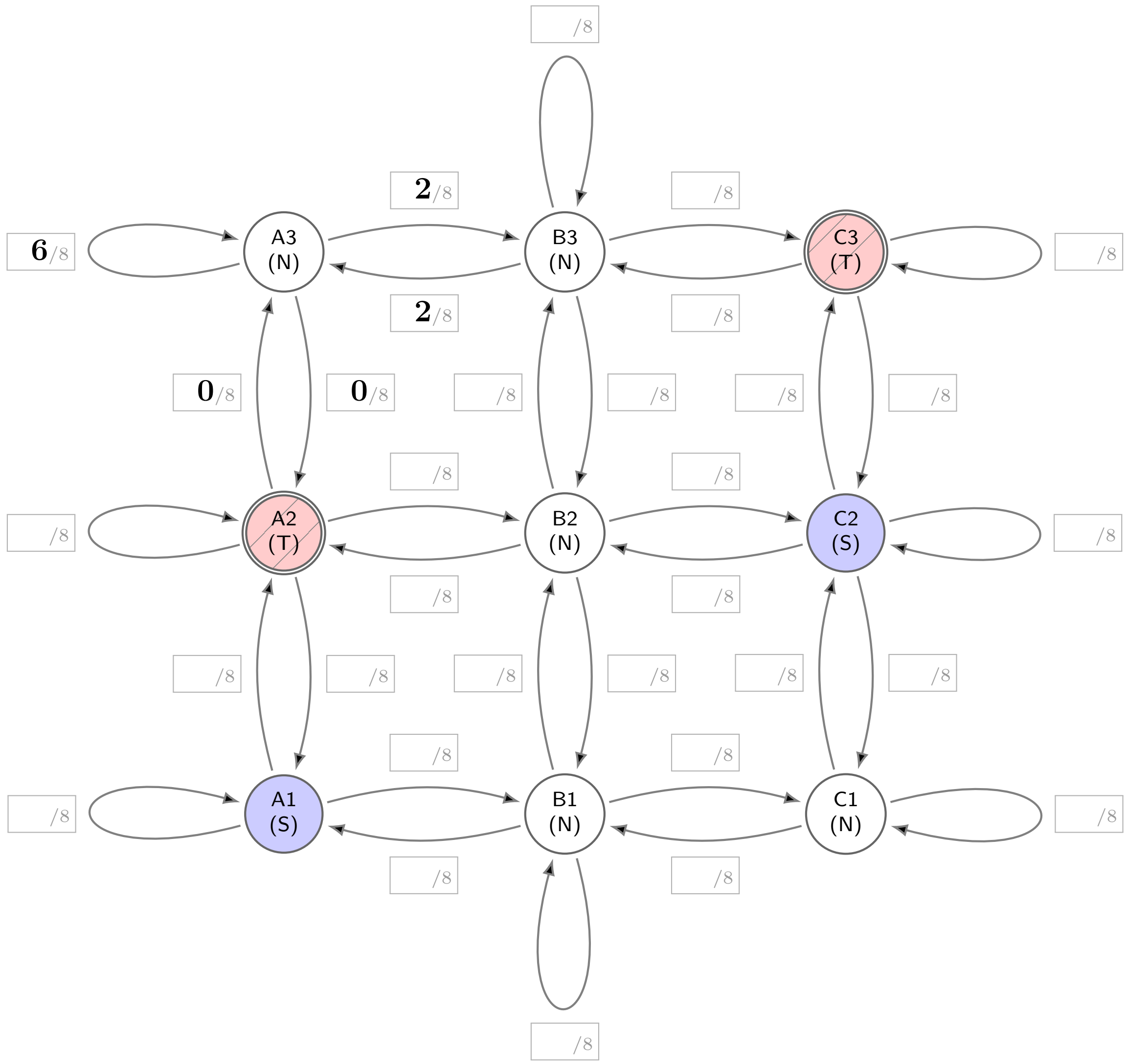
   - The map is a 3×3 grid of tiles with rows $\in \{1, 2, 3\}$ and columns $\in \{A, B, C\}$.
   - You start at $t = 0$ on a random tile X_0. Each tile is equally likely.
   - At every time step, the player moves in a random direction that depends on the type of tile they are in. There are three types of tiles:
     - Normal (**N**): Equal probability of moving *up, down, left* or *right.* All unlabeled tiles are normal.
     - Stuck (**S**): There is a 50% chance of not moving. Otherwise, the player moves randomly like **N**.
     - Trap (**T**): The player never moves from the trap state once entered.
   - **Walls:** If you attempt to move into a wall (a thick black line), you stay in your current square for that time step instead of moving.
   - **Observations:** You can see only the type of tile you are standing on (**N, S,** or **T**). Your observation of your tile type at X_t is O_t.

Room



Assume now that we are in the room above. Note that there are walls on three sides of the **T** in tile **2A**, meaning a player can only enter that **T** tile from location **2B**, but the **T** in tile **3C** can be entered from **B3** or **C2**. *Note: None of the problems in this section require matrix multiplication to solve.*

(a) (10 points) Finish the HMM transition diagram that represents the room by drawing all the other transitions between nodes and their respective probabilities. **We have already filled in the denominator of 8 for all probabilities, so so only write numerator**. For example, the probability of transitioning from A3 to A3 would be $\frac{6}{8}$, so we would write 6 on this diagram. We have already done all the transitions to and from A3 for you. Fill in the rest of the diagram.



(b) (2 points) At $t = \infty$ what is the probability you have fallen into a trap?

(c) You have survived for two turns in the room and experience the following observation sequence:

$$O_0 = \mathbf{S}, \quad O_1 = \mathbf{S}, \quad O_2 = \mathbf{N}$$

i. (2 points) What is your belief state after O_0? That is, $P(X_0 \mid O_0 = \mathbf{S})$?

ii. (2 points) And then, after you take another step? That is, $P(X_1 \mid O_0 = \mathbf{S}, O_1 = \mathbf{S})$?

iii. (2 points) And then, after you take another step? That is, $P(X_2 \mid O_0 = \mathbf{S}, O_1 = \mathbf{S}, O_2 = \mathbf{N})$?

iv. (3 points) What is $P(X_1 \mid O_0 = \mathbf{S}, O_1 = \mathbf{S}, O_2 = \mathbf{N})$?

v. (3 points) What is $P(X_0 \mid O_0 = \mathbf{S}, O_1 = \mathbf{S}, O_2 = \mathbf{N})$?

(d) (10 points) What is the **most likely sequence** of states (path) X_0, X_1, X_2 that explains these observations? If there are multiple equally likely paths for the observations $O_0 = \mathbf{S},\ O_1 = \mathbf{S},\ O_2 = \mathbf{N}$, list one of them.

End of exam.