1 MCTS

1.1 MCTS Review

Upper-confidence bound The UCB formula is:

$$\text{UCB}(N, N_k, U_k) = \begin{cases} \frac{U_k}{N_k} + C\sqrt{\frac{\log N}{N_k}} & \text{if } N_k > 0 \\ \infty & \text{otherwise} \end{cases}$$

Here, N is the visitation count to the parent node; N_k is the visitation count to a child node; U_k is the total utility accumulated at the child.

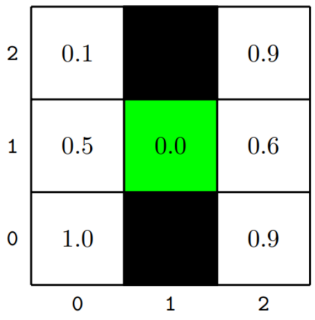
MCTS with UCB

1. **Selection** Starting at the root of the search tree, choose moves down to a leaf node according to UCB. Return the leaf node.
2. **Expansion** Grow the search tree from the leaf node by generating all its children. If the leaf node is already at the end of the horizon, skip this step.
3. **Simulation** Perform Monte-Carlo playout from the leaf node. Note that no new node is added to the search tree.
4. **Back-propagation** Use simulation's result to update all visitation counts and cumulative rewards, going up to the root.

1.2 MCTS Practice

Consider the reward maximization problem in the grid on the right.

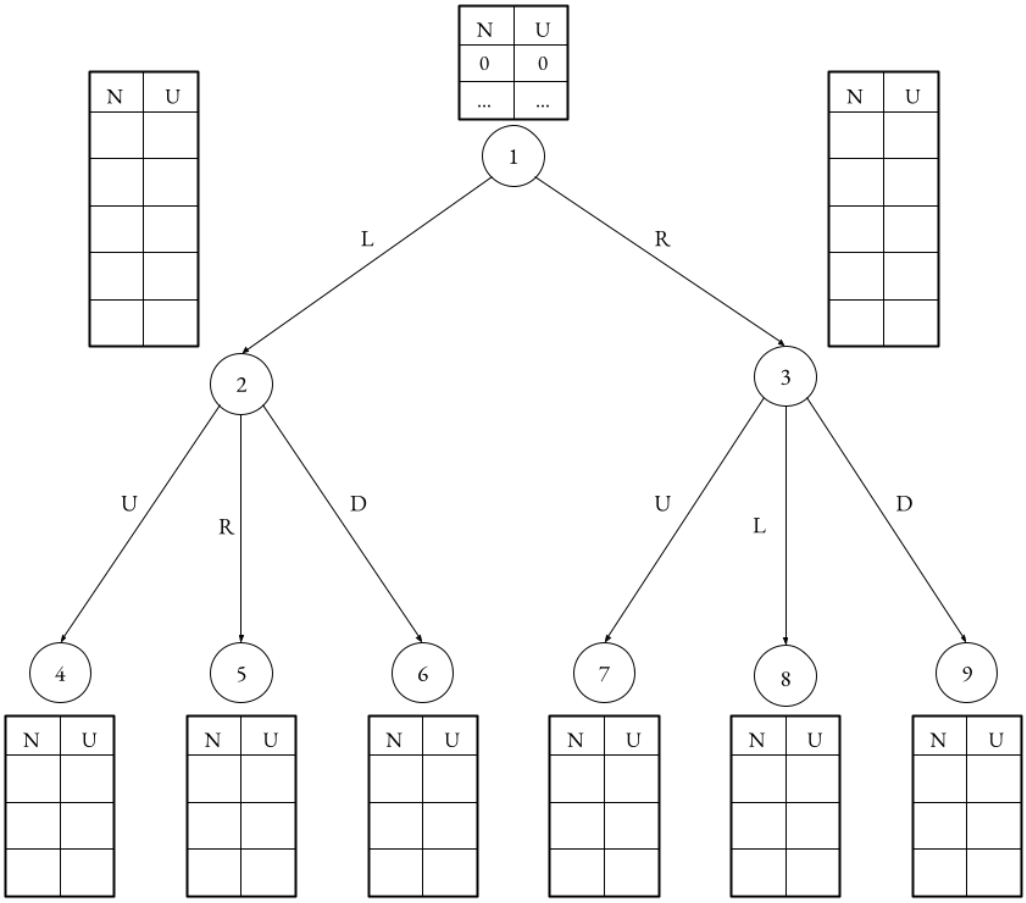
- The agent starts at s_0 at the center
- The agent can move left, right, up, or down, except that it cannot move onto black grid cells
- Entering each grid cell yields a reward (specified within the box)
- Assume a horizon of 2
- Assume that $C = 1.0$



Simulate running MCTS and fill in the search tree below until you run out of cells (note: some table cells may remain empty). Assume that the random choices are $[0, 2, 1, 2]$ (select the first child $\rightarrow$ the third child $\rightarrow$ the second child $\rightarrow$ the third child). Break ties during the selection phase by selecting the leftmost child in the tree.

The following computations might be helpful:

- $\sqrt{\frac{\log(n)}{1}} - \sqrt{\frac{\log(n)}{n-1}} \leq 0.5$ for $2 \leq n \leq 4$
- $\sqrt{\frac{\log(n)}{2}} - \sqrt{\frac{\log(n)}{n-2}} \leq 0.5$ for $3 \leq n \leq 8$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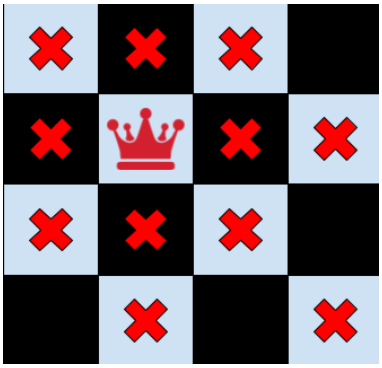
2 CSP

2.1 CSP Review

1. **Constraint Graph** Graph where nodes correspond to variables of a problem and edges connect any two variables participating in a constraint.
2. **Arc Consistency** A variable in a CSP is arc-consistent if every value in its domain satisfies the variable's constraints. A graph is arc-consistent if every variable is arc-consistent with every other variable

2.2 CSP Practice: 4-Queens problem

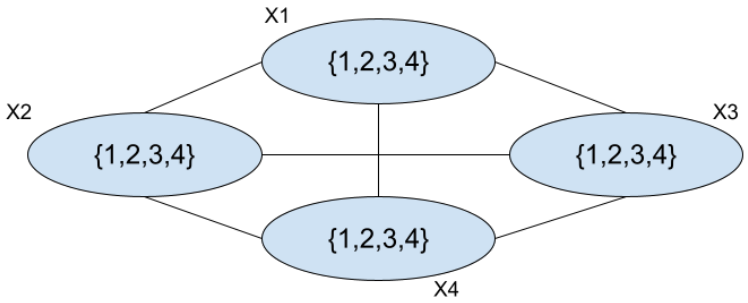
Consider a 4x4 chess-board. Place four queens on the board such that no two of the queens can attack each other. Queens can attack other queens placed in the same row, column, or along the same diagonal



For example, if we place a queen on (2,2), the following squares can not be occupied by a queen.

Since we know that each column can only have one queen, let us represent this problem with four variables, X_1, X_2, X_3, X_4, where X_i represents the row of the queen in column i. In the example above, $X_2 = 2$. The domain of each X_i is $\{1, 2, 3, 4\}$.

We can represent this using the following constraint graph. Each edge in the graph represents a constraint that the two queens connected by the edge may not be in the same row and that two queens may not be along the same diagonal.



2.2.1 Problem 1

Execute the AC-3 algorithm with the additional constraint that $X_1 = 1$. Does a solution exist? How can you tell?

2.2.2 Problem 2

Execute backtracking with forward-checking to find a solution to the above CSP. Assign variables in the order X_1, X_2, X_3, X_4 and the values from 1 to 4.