# 6.411/16.420 Notation Glossary

*Last updated: August 20, 2022*

## General

- Sets: $\mathcal{S}, \mathcal{X}$, etc.
- Functions: $f : \mathcal{S} \to \mathcal{X}$, $g : \mathcal{S} \times \mathcal{X} \to \mathcal{S}$, etc.; capitalization also allowed

## Constraint Satisfaction

- $\mathcal{X}$ is a set of variables
- $X_i$ is a variable
- $\mathcal{D}$ is a set of domains
- $D_i$ is the domain of variable $X_i$: that is, the discrete set of values it can take on
- $\mathcal{C}_i$ is constraint, which specifies a tuple of variables (subset of $\mathcal{X}$) and a relation on those variables (subset of the Cartesian product of the domains of the variables)

## Logic

- $\alpha$ (Greek letters, in general): a logical sentence
- $M(\alpha)$ : the set of all models of $\alpha$ (possible worlds in which $\alpha$ is true)
- $\alpha \models \beta$ : $\alpha$ entails $\beta$: in every model (possible world) in which $\alpha$ is true, $\beta$ is also true
- $\alpha \equiv \beta$ : $\alpha \models \beta$ and $\beta \models \alpha$ (the set of possible worlds where $\alpha$ is true is the same as the set of possible worlds where $\beta$ is true)
- $\alpha \vdash \beta$ : $\beta$ can be proved from $\alpha$
- $\mathrm{KB}$ : "knowledge base" – generally a set of sentences that is assumed to be true
- $\neg$ : not
- $\vee$ : or
- $\wedge$ : and
- $\Rightarrow$ : implies
- $\Leftrightarrow$ : if and only if
- $true, false$ : truth values

- $\mathit{True}, \mathit{False}$ : atomic sentences
- $\exists$ : existential quantifier : "there exists"
- $\forall$ : universal quantifier : "for all"
- $P$ : In propositional logic, a proposition symbol, which may have the value $\mathit{true}$ or $\mathit{false}$. In first-order logic, could be a constant, or a predicate or function symbol.
- $x$ : In first-order logic, a variable.
- $\frac{\alpha \quad \beta}{\gamma}$ is an inference rule that allows you to conclude $\gamma$ if you have already proved $\alpha$ and $\beta$.
- $\{A \to 4, R \to >, x \to \clubsuit\}$ is an *interpretation*, mapping *constant symbol* $A$ to 4, *predicate symbol* $R$ to the greater-than relation, and variable $x$ to another entity, $\clubsuit$.
- $\{v/B, u/v, x/F(y), y/F(G(C))\}$ is a *substitution*. Note that a *substitution* maps syntactic symbols (variables) into other syntactic expressions (terms) while a *interpretation* maps syntactic symbols (variables, constant symbols, function symbols, predicate symbols) into *actual real-world entities* like people or tables or integers or relations.
- A *term* is a syntactic expression that can refer to something. It can be: a constant symbol, a variable, or a function applied to 1 or more terms.
- An *atom* is a syntactic expression that has a truth value, but doesn't involve quantifiers or Boolean connectives. It can be: a proposition symbol or a predicate symbol applied to 1 or more terms.
- A *literal* is an atom or a negated atom.

# Probability

- $x$ is a scalar variable with a specific value that is not given
- $X$ is a random variable
- $X = x$ denotes that the random variable has a specific value
- $\mathbf{x}$ is a vector-valued variable
- $\mathbf{X}$ is a vector-valued random variable
- $P(X)$ is the probability distribution of the random variable $X$. It is a function for continuous variables or a probability mass function (most likely given as a table) for discrete variables.
- $P(x)$ is the probability of the specific value of $x$ for discrete variables, or the relative likelihood of a specific value of $x$ for continuous variables. It is a scalar quantity.
- $P(X \leq 0)$ is the probability that $X$ is less than 0. It is a scalar quantity.
- $f_X(x)$ is the relative likelihood of the value $x$ under the cumulative distribution of $X$.
- $P(X, Y)$ is the joint distribution of $X$ and $Y$
- $P(X \mid Y)$ is the conditional distribution of $X$ given $Y$
- $P(X; \theta)$ is a distribution over $X$, where the distribution has parameters $\theta$ (e.g., mean and variance). Note that $\theta$ is not a random variable, and there is no implied $P(\theta)$.

- $P(X \mid \theta)$ is the conditional distribution over $X$, given $\theta$. Note[1] that in this case, $\theta$ is a random variable, and there is an implied distribution $P(\theta)$.
- $\rho(X, Y)$ is the correlation between and $X$ and $Y$
- $E[X]$ is the expected value of $X$
- $E_X[f(X)]$ is the expected value of $f(X)$ with respect to the distribution over $X$
- $X \sim D$ means that random variable $X$ is distributed according to distribution $D$.
- $X \sim N(\mu, \sigma^2)$ denotes the random variable $X$ is distributed according to a normal distribution with parameters mean $\mu$ and variance $\sigma^2$
- $\mathbf{X} \sim N(\mu, \Sigma)$ denotes that the random variable $\mathbf{X}$ is distributed according to a multivariate normal distribution with parameters mean vector $\mu$ and covariance matrix $\Sigma$
- $\hat{\mathbf{x}}$ is the estimate of unknown $\mathbf{x}$
- $\bar{\mathbf{x}}$ is the sample mean of samples drawn from $P(\mathbf{X})$

# Heuristic Search and Classical Planning

- Classical planning model (GB): $\mathcal{S} = \langle S, s_0, S_G, A, f, c \rangle$
  - $S$ is a finite set of states
  - $s$ is a state, $s_0$ is the initial state
  - $S_G$ is a nonempty set of goal states
  - $A$ is a set of actions
  - $A(s)$ is a set of applicable actions for state $s$
  - $f(s, a)$ is the deterministic transition (successor) function, $s' = f(s, a)$
  - $c(s, a)$ is the positive cost of doing action $a$ in state $s$
- Graph Search: $G = (V, E)$ with $V \subset S$ and $E = (s, s')$, $s' = f(s, a)$ for all relevant $s$ and $a$.
  - $f(n)$ is the estimated cost of cheapest solution through $n$
  - $g(n)$ is the path cost to reach $n$
  - $h(n)$ is the estimate of cost to go from $n$ to goal
  - AIMA: $c(n, a, n')$ is the cost of successor of $n$ generated by action $a$

# MDPs

- Finite-horizon MDP: $M = (\mathcal{S}, \mathcal{A}, R, P, H)$ where
  - $s \in \mathcal{S}$ is a state
  - $a \in \mathcal{A}$ is an action
  - $R : \mathcal{S} \times \mathcal{A} \times S \to \mathbb{R}$ is a reward function
  - $P(S_{t+1} = s' \mid S_t = s, A_t = a)$ is the transition distribution
  - $H \in \mathbb{Z}^+$ is a horizon

[1]The Kochenderfer et al book unfortunately does not distinguish between $P(X \mid \theta)$ and $P(X; \theta)$.

- Infinite-horizon MDP: $M = (\mathcal{S}, \mathcal{A}, R, P, \gamma)$
  - $\gamma \in [0, 1)$ is a temporal discount factor
  - May also include $\mathcal{D} \subseteq \mathcal{S}$, a set of "done" (aka sink, absorbing, terminal, goal) states
  - Others are same as above
- Policy: $\pi : \mathcal{S} \to \mathcal{A}$
- On-line vs. off-line planning
  - Off-line: A plan or policy is computed ahead of time, and never changed.
  - On-line: The plan or policy may be recomputed during execution.
- Open-loop vs. closed-loop
  - Open-loop: A plan (a single sequence of actions) is executed as-is, regardless of the states encountered during execution.
  - Closed-loop: A policy (map from states to actions) is used; execution depends on the states encountered.

# POMDPs

- $b$ : belief state, which is a probability distribution over underlying state space $\mathcal{S}$
- $b(s)$ : probability (or density) assigned to state $s \in \mathcal{S}$ by belief $b$.
- Infinite-horizon POMDP: $M = \langle \mathcal{S}, \mathcal{A}, \mathcal{O}, R, T, O, \gamma \rangle$ which is an MDP augmented with
  - $o \in \mathcal{O}$ is an observation
  - $O(o \mid a, s') = P(O_{t+1} = o \mid A_t = a, S_{t+1} = s')$ is the observation distribution